Filter Morphing for Audio Signal Processing

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Abstract

The coefficient encoding scheme called "ARMAdillo" for a second order direct form digital filter was previously proposed [1]. In this paper we examine the behavior of a second order digital filter, whose coefficients are encoded using the ARMAdillo scheme, when it morphs from one frame to another. We restrict our discussion to biquadratic parametric equalizers and shelving filters. Results show that using encoded coefficients, the morphing performance of a parametric equalizer or shelving filter is far superior to the non-encoded case.

1 Introduction

Digital filters are playing a more and more important role in audio signal processing. Very often, digital filters are designed off line and their parameters remain unchanged during use. However, in many cases, such as in a live performance or in the post-processing of music sounds, continuous control of filter parameters is required. In other words, the parameters of the filter, and thus its coefficients must be updated continuously. This kind of time-varying digital filter also has applications in spectral modeling, sound analysis and synthesis, and in many other aspects of audio signal processing. In [2], the authors proposed that a set of fixed second order resonant filters be used to get short-duration excitation signals for sounds of percussive instruments. It is well known that even for percussive sounds, the resonance parameters, i.e. the frequencies and damping factors of exponentially damped sinusoids are slightly changing in time. Thus, if time-varying resonant filters could be used in place of steady ones, greater similarity and lower data rate of the short-duration excitation signals would be expected. The problem is that in a digital implementation, intolerable errors, instabilities and undesired audible artifacts can occur when filter parameters are updated. Preventing these effects from happening is a challenge for the design and implementation of a time-varying digital filter. Furthermore, it is desired in some scenarios that the change of filter parameters be made to correspond properly to the auditory perceptual parameters. In the case of a parametric equalizer, this requires that the center frequency be changed logarithmically from one to another while the filter gain changes linearly on a decibel scale.

Design and implementation of a time varying digital filter meeting the above requirements has received great attention from many researchers/engineers in the audio engineering community. Reference [3] discussed the cause and conditions of audibility of distortions in implementing time-varying digital filters and pointed out that the audibility is a function of the input signal, the filter parameters being changed, their rate of change, and the masking properties of the human ear. In [4], the authors offered practical strategies for updating filter parameters to minimize audible artifacts. These discussions are very useful for further research of time-varying filter design.

The requirements for real-time update of filter parameters can result in a significant computational burden. The computational complexity can increase the cost of the system dramatically. In [6], a system based on a personal computer with a DSP subsystem is used for real-time control of parameters of parametric equalizers. Apparently, this much computational power is not realistic for an economical electronic musical instrument. In music applications, we are more interested in the real-time control of filter parameters at a speed as fast as the sampling rate (one order faster than that reported in [6]) with very low cost. Therefore, according to the discussion in [3], a key issue of our concerns becomes preserving the features of the frequency response of the filter when it is morphed from one set of parameters to another.

Considering the difficulty and complexity of the problem, we restrict our discussion to the filter morphing of parametric equalizers and shelving filters. We have found that by encoding the filter coefficients of a second order filter and linearly interpolating them, the problem of efficiently updating filter parameters and yet preserving the features of its frequency response

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can be solved satisfactorily to some extent. The encoding scheme we use is called ARMAdillo and was proposed previously in [1]. We briefly discuss the filter structure for implementing a parametric equalizer or shelving filter in the next section. We then explain the idea of the ARMAdillo encoding scheme in detail in Section 3. Finally, in Section 4, we present and discuss our experiment results.

2 Filter Structures

Parametric equalizers and shelving filters are two of the most common filters in audio signal processing. They are widely used in digital mixing desks, electronic musical instruments, and in studios for single frequency attenuation/boost for signal shaping and enhancement or controlling the gain within a number of specified frequency bands. For parametric equalizers, a few different second order IIR filter topologies have been proposed (see [5] and the references therein). These include the direct form and all-pass with feedforward. Although they have different round-off noise, limit cycle, and saturation characteristics, all can be simplified to have the same form of transfer function as given in (1), having 5 independent coefficients.

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}. \tag{1}$$

Since our encoding scheme is based on a direct implementation of a second order transfer function, we will only consider the filter structure (topology) of the direct form for both the parametric equalizer and the shelving filter. Note that a slight modification of (1) is

$$\begin{aligned} H(z) &= b_0 \frac{1+\frac{b_1}{b_0}z^{-1}+\frac{b_2}{b_0}z^{-2}}{1+a_1z^{-1}+a_2z^{-2}} \\ &= c_0 \frac{1+c_1z^{-1}+c_2z^{-2}}{1+a_1z^{-1}+a_2z^{-2}}, \end{aligned} \tag{2}$$

where $c_0 = b_0$, $c_1 = \frac{b_1}{b_0}$ and $c_2 = \frac{b_2}{b_0}$. The filter structures corresponding to (1) and (2) are shown in Figure 1.

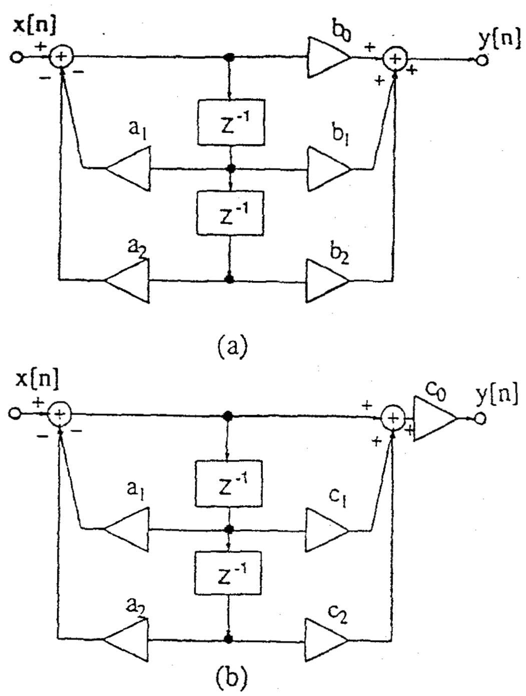

Figure 1: Two slightly different direct form filter structures

3 The ARMAdillo Encoding Scheme

The ARMAdillo encoding scheme is to be used to encode the coefficients a_i and c_i, $i = 1,2$ in (2). It aims to decouple the control of the radius and angle of a pole or zero such that the radius and angle of a pole or zero can be independently controlled by two coefficients a_1 and a_2 or c_1 and c_2, respectively. In the meantime, we would like the resonant peak or notch in the magnitude response produced by a pole or zero to change linearly on a decibel scale while the resonant frequency changes logarithmically when the coefficients are dynamically swept.

Since the encoding scheme applies to the coefficients of both the numerator and the denominator of (2) we use the following general polynomial for discussion,

$$p(z) = 1 + t_1 z^{-1} + t_2 z^{-2}. \tag{3}$$

It is well known that if the above polynomial has complex conjugate roots, $r_{1,2} = \rho e^{\pm j\theta}$, (these roots are nothing but the poles or zeros of the transfer function (2)), we have

$$t_1 = -2\rho\cos\theta \quad \text{and} \quad t_2 = \rho^2. \tag{4}$$

Noting that the height of the magnitude response at the resonant frequency is approximately inversely proportional to the distance from the root to the unit circle, we can relate this height h in dB to the root radius ρ, i.e.

$$h = 20\log_{10}\frac{1}{1-\rho}. \tag{5}$$

Since $t_2 = \rho^2$, if ρ (or t_2) is close to 1, and we encode t_2 into v_2 as follows

$$t_2 = 1 - t_2' = 1 - 2^{-v_2} \quad \text{or} \quad t_2' = 1 - t_2 = 2^{-v_2}, \tag{6}$$

then [1, 7]

$$h \approx 6v_2 + 6. \quad (7)$$

That is, h will be linear in v_2.

From (4), we see that the ρ only depends on t_2. But θ is determined by both t_1 and t_2. Using the approximation [7]

$$t_1 \approx -2\cos\theta + t_2', \quad (8)$$

let $u = -2\cos\theta$, noting that u takes values in the range of $[-2,\ 2]$ for $\theta \in [0\ \pi]$, and let

$$t_1' = \frac{u+2}{4} \quad \text{or} \quad u = -2 + 4t_1'. \quad (9)$$

Then, t_1' will take values in the same range of $[0,\ 1]$ as t_2'. Thus, we can encode t_1' into v_1 in the same manner as for t_2'. Combining equations (8) and (9), we get

$$t_1 = -2 + 4t_1' + t_2' = -2 + 4 \cdot 2^{-v_1} + 2^{-v_2}. \quad (10)$$

We then can obtain [1, 7]

$$\Omega \approx -\frac{1}{2}v_1 + \log_2 \frac{f_s}{20\pi}, \quad (11)$$

where f_s is the sampling rate and Ω is a quantity called "musical octave number" defined as $\Omega = \log_2 \frac{\theta f_s}{40\pi}$.

The above equation (11) shows that v_1 is approximately linear in musical octave number and thus indeed logarithmic in θ (the peak/notch frequency of the system) as desired.

4 Experiments and Discussions

It is well known that for the transfer function of a biquad shown in (1) or (2) the peak or notch of its magnitude frequency response is primarily determined by its poles or zeros, respectively. We have shown how to encode the coefficients of such a biquadratic transfer function so that the height of its peak or notch changes in a decibel scale and the peak or notch frequency changes logarithmically if we linearly morph the encoded coefficients associated with one set of filter parameters to another. Being able to do this is extremely important for musical sound manipulation and processing since the parameter change of a resonant filter can be made according to perceptual units. In practice, an octave shifting encoding scheme can be used to approximate the inverse of the exponential operation required in (6) and (10), and thus the exponential decoding can be reduce to a few logic gates and a barrel shifter in a VLSI implementation. The morphing (interpolation) of encoded filter coefficients involves only two multiplications and an addition for one coefficient. Therefore, the whole operation for updating filter parameters can be done at a speed as fast as the sampling rate. This is very attractive to many musicians and electronic musical instrument users. It should be noticed that a straightforward way of updating filter parameters at a comparable speed is to interpolate filter coefficients directly (without encoding, called direct interpolation), but, as we will see, the performance is not satisfactory. In the following examples, we use the filter structure (a) in Figure 1 for the direct interpolation. To employ the ARMRdillo encoding scheme the filter structure (b) in Figure 1 is used. In this case, only a_i and c_i, $i = 1, 2$ are encoded and linearly interpolated, and c_0 is directly interpolated. In all examples, the sampling rate is 44.1 kHz.

Figure 2 demonstrates the filter morphing performance of a parametric equalizer obtained by linearly interpolating two sets of encoded coefficients which correspond to the parametric equalizers with center frequency at 80 Hz and 10 kHz, respectively. Their bandwidth is 2 semitones and gain is 15 dB. For the same range of frequency change, the performance of the direct interpolation is shown in Figure 3. From these figures, we see that the morph of the parametric equalizer with coefficient encoding performs reasonably well in a wide range of frequency change. The center frequencies of the equalizer are uniformly located on a logarithmic frequency axis. However, the center frequencies of direct interpolation are cluttered at the high frequency end even though the shape of the parametric equalizer is, surprisingly, preserved perfectly. Figures 4 and 5 show the coefficient trajectories for poles corresponding to Figures 2 and 3 which are compared with the theoretic values, i.e. the calculated coefficients of the parametric equalizer when its center frequency changes. These figures show that for a parametric equalizer with narrow bandwidth, the logarithmic change of its center frequency can be well approximated by uniformly sweeping the encoded coefficients. Figures 6 and 7 are another example of a parametric equalizer where both its center frequency and gain are changed. It should be noted that the asymmetry of the shape with coefficient encoding increases when the bandwidth gets wider. This is partly due to the fact that when the bandwidth of a parametric equalizer increases, its zeros (for boost) or poles (for cut) leave the unit circle, resulting in a poor approximation of equation (8).

From Figures 3 and 7, it appeared that if an exponential morphing was used, the direct interpolated equalizers would be uniformly located on a logarithmic frequency axis. Unfortunately, it is not the case due to the coupling between coefficients. In addition, we found that if the filter structure (b) had been used for direct interpolation, the shape of the parametric equalizers would not have been preserved. The precise relationship between the shape of the magnitude response and the filter structure is still under investigation. This relationship may or may not exist in general.

Finally, we give an example of filter morphing of a shelving filter which is shown in Figures 8 and 9. We see that, by using coefficient encoding, the low shelving filter with 30 dB gain and 100 Hz cut-off frequency nicely morphs to the low shelving filter with 10 dB gain and 1 kHz cut-off frequency while the performance of the direct interpolation is not acceptable.

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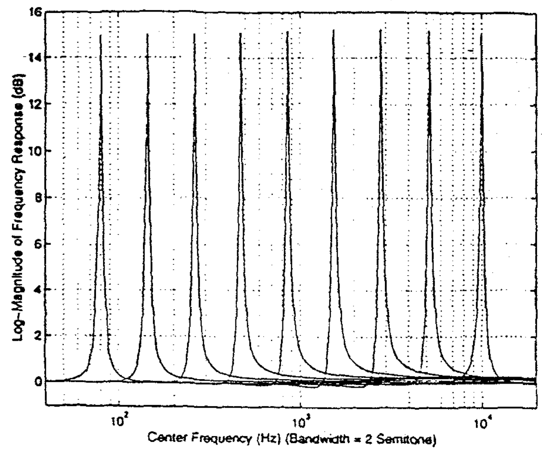

Figure 2: Filter morphing of a parametric equalizer with coefficient encoding

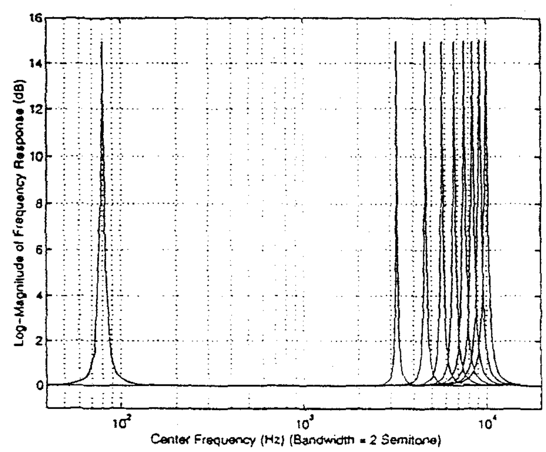

Figure 3: Filter morphing of a parametric equalizer with direct interpolation

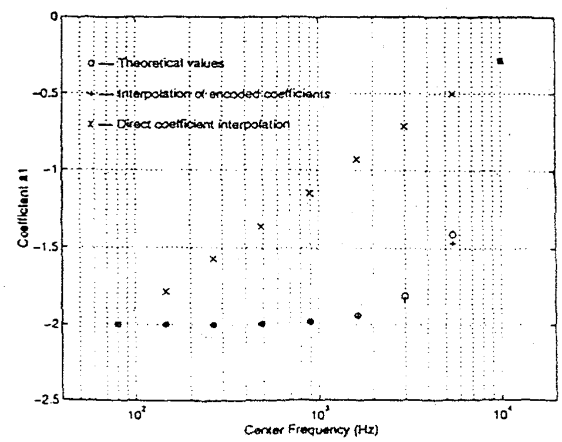

Figure 4: Coefficient trajectories for a_1

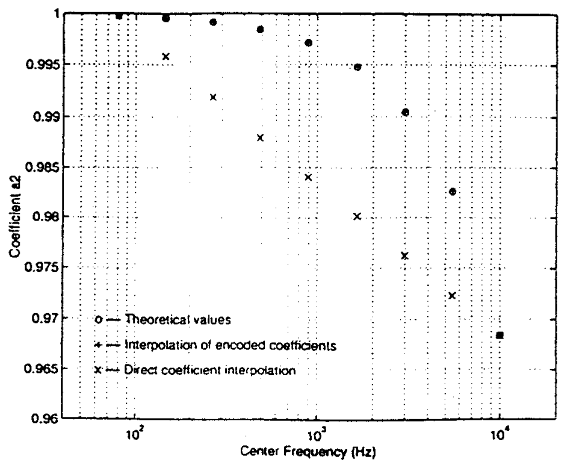

Figure 5: Coefficient trajectories for a_2

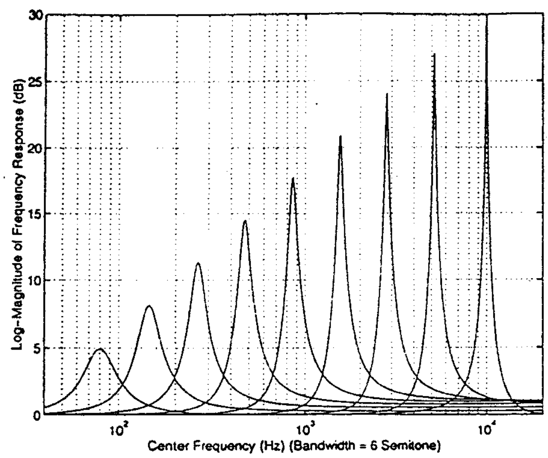

Figure 6: Filter morphing of a parametric equalizer with coefficient encoding

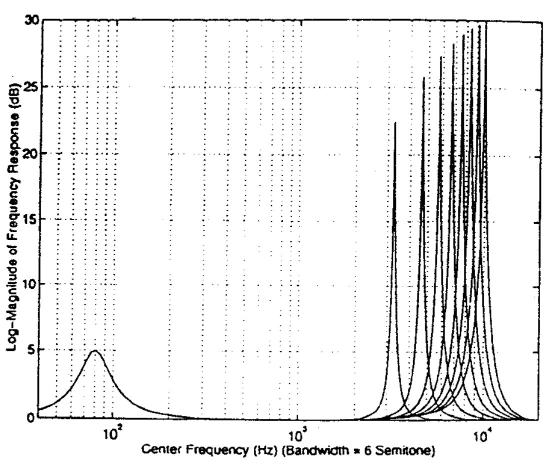

Figure 7: Filter morphing of a parametric equalizer with direct interpolation

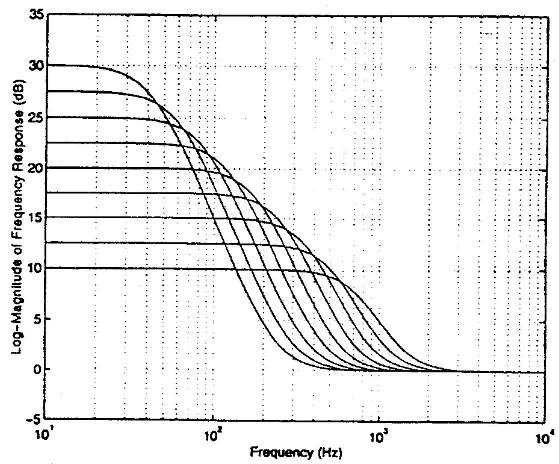

Figure 8: Filter morphing of a shelving filter with coefficient encoding

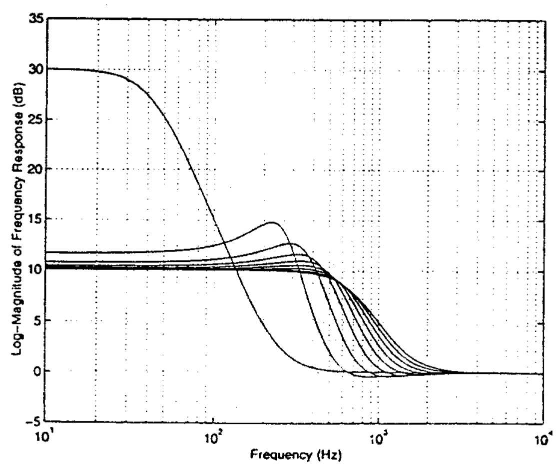

Figure 9: Filter morphing of a shelving filter with direct interpolation